Heavy-Quarkonium Collider Physics

Outline

- The Importance of Heavy-Quarkonium Physics
- Heavy Quarkonium: Progress, Puzzles and Opportunities
- Gluon Fragmentation to a color-singlet $Q\bar{Q}$ pair in order v^4
- Factorization Theorems for Exclusive Quarkonium Production

The Importance of Heavy-Quarkonium Physics

- A useful theoretical laboratory for understanding the interplay between perturbative and nonperturbative QCD.
  - The heavy-quark expansion gives better theoretical control over nonperturbative effects.
  - Potential models are valid.
  - Fock-state expansion is expansion is well controlled.
- Insights gained in studying heavy quarkonium will likely be important in other areas.
  - Surprising enhancements of NLO (NNLO) cross sections by an order of magnitude compared to LO (NLO) cross sections.
  - All-orders resummation of the velocity expansion may have implications for resummation of higher-twist effects in light-hadron processes.
- There is a great deal of activity in heavy-quarkonium in collider experiments.
  - CDF, D0, Belle, BESII, ALICE, ATLAS, CMS, LHCb, PHENIX, STAR all have active programs in heavy-quarkonium physics.
  - Already 47 papers on quarkonium physics have been written by the LHC experiments. Many more LHC results to come.
- We should take advantage of the wealth of experimental information to learn more about QCD.

Heavy Quarkonium: Progress, Puzzles and Opportunities

Nora Brambilla (TU München), G.T. Bodwin (ANL), *et al.*

Eur. Phys. J. C71, 1534 (2011)

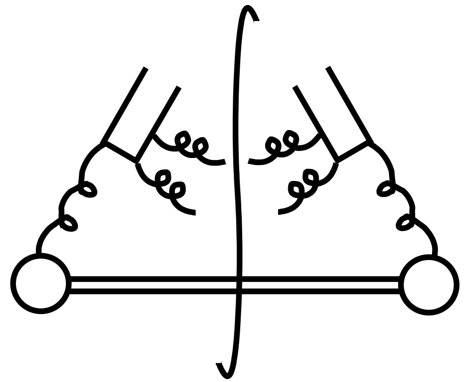
- Members of the Quarkonium Working Group (QWG) have prepared a comprehensive (181 page) document that describes recent progress in quarkonium physics and the outstanding current issues in experiment and theory.
- The document also summarizes new opportunities in quarkonium physics at present and future facilities.
- It is an important resource for the collider experimental programs, especially at the LHC.
- Topics covered are spectroscopy, decay, production, production in media, and the experimental outlook.
- GTB was a coordinator and principal author of the section on production.

Gluon Fragmentation to a color-singlet $Q\bar{Q}$ pair in order v^4

GTB (ANL), U-Rae Kim (Korea U.), and Jungil Lee (Korea U.)

- Why is order v^4 important?
  (v is the heavy-quark velocity in the quarkonium CM frame.)
  - Gluon fragmentation to a color-octet $Q\bar{Q}$ pair is thought to be the dominant J/ψ production mechanism at large p_T.
  - But the color-octet contribution by itself is not the complete story.
  - Gluon fragmentation to a color-singlet $Q\bar{Q}$ pair at order v^4 is connected to gluon fragmentation to a color-octet $Q\bar{Q}$ pair through a logarithm of the factorization scale.
  - Suggests that gluon fragmentation to a color-singlet $Q\bar{Q}$ pair may be important at order v^4.
  - Only the sum of the octet and singlet contributions is independent of the factorization scale.

- The calculation is technically challenging.
  - Four partons in the final state.
  - Two real gluons potentially produce double IR divergences.
  - Angular integrals in the fragmentation function are over only the transverse directions—awkward.



  - Four derivatives with respect to the external momenta produce an explosion of terms and lead to double IR divergences.
  - This is the first NRQCD calculation involving two-loop renormalization of NRQCD operators.
- Strategy of the calculation
  - Remove divergences by making infrared subtractions.
    Construct subtractions using a new scaling method instead of dipole subtractions.
  - Evaluate the finite part numerically.
  - Evaluate the subtractions analytically in dimensional regularization.
  - Remove the single and double poles in ϵ by absorbing them into ${}^3S_1$ and ${}^3P_J$ color-octet NRQCD matrix elements.
  - Requires calculation of the order-ϵ terms in the ${}^3S_1$ and ${}^3P_J$ color-octet short-distance coefficients.
- Work on this calculation is nearing completion.

Factorization Theorems for Exclusive Quarkonium Production

G.T. Bodwin (ANL), J. Lee (Korea U.), X. Garcia i Tormo (ANL, U. of Alberta)

Phys. Rev. Lett. **101**, 102002 (2008)

Phys. Rev. D **81**, 114014 (2010)

Based on work on factorization theorem for light mesons.

Phys. Rev. D **81**, 114005 (2010).

Factorization in Hard-Scattering Processes

- Factorization theorems are the theoretical foundation for predictions in perturbative QCD.
- The goal in factorization is to separate
  perturbative processes at the scale of the large momentum transfer Q
  nonperturbative processes at the scale of Λ_{QCD} or smaller.
- The perturbative contributions are contained in the short-distance coefficients.
  - The short-distance coefficients are process dependent.
  - The short-distance coefficients can be calculated in perturbation theory.
- The nonperturbative contributions are contained in long-distance quantities, such as parton distribution functions, NRQCD matrix elements, light-cone distributions.
- The predictive power of factorization relies on the universality (process independence) of the long-distance quantities.

Why Consider Factorization Theorems for Heavy-Quarkonia?

- No explicit proofs of factorization existed for heavy-quarkonium production—only conjectures.
- An apparent problem with factorization was noticed in B-meson decays to P-wave quarkonium in NLO [Song, Meng, Gao, Chao (2003)].
- Nothing definite was known about the possible corrections to heavy-quarkonium factorization formulas.

Statements of the Theorems

- We proved factorization formulas for two exclusive processes:
  - $e^+e^- \longrightarrow$ charmonium $+$ charmonium,
  - $B \longrightarrow$ light meson $+$ charmonium.
- These are the first factorization theorems to be proven for quarkonium production.

$e^+e^- \longrightarrow$ charmonium $+$ charmonium

- The amplitude has the factorized form

$$\mathcal{A}\left(e^+e^- \to \gamma^* \to H_1 + H_2\right) = \sum_{ij} A_{ij} \langle H_1|\mathcal{O}_i|0\rangle \langle H_2|\mathcal{O}_j|0\rangle \,.$$

  $\langle H_n|\mathcal{O}_i|0\rangle$ is an NRQCD matrix element.
  A_{ij} is a short-distance coefficient.

- Holds to all orders in α_s up to corrections of order $(m_c v^2)^2/s$ for e^+e^- annihilation to two S-wave charmonia.

$B \longrightarrow$ light meson $+$ charmonium

- The amplitude has the factorized form

$$\mathcal{A}\,(B \rightarrow H_1 + K) = \sum_{ife} F^{B\rightarrow K}_{f(i,e)}(M_1^2) A_{ie} \langle H_1 | \mathcal{O}_i | 0 \rangle + \sum_{ije} A'_{ije} \otimes \Phi_{Kj} \otimes \Phi_{B1} \langle H_1 | \mathcal{O}_i | 0 \rangle .$$

$F_f^{B\rightarrow K}$ is a B-meson to light-meson form factor.
Φ_{Kj} is a light-meson light-cone distribution amplitude.
Φ_{B1} is a B-meson light-cone distribution amplitude.
A_{ie} and A'_{ije} are short-distance coefficients.

- Similar to the factorized form for B-meson decay to two light mesons [Beneke, Buchalla, Neubert, Sachrajda (2000)].

- Holds to all orders in α_s up to corrections of order $m_c v^2/m_b$ for B-meson decays to an S-wave charmonium.

Sketch of the Proof

(for $e^+e^- \longrightarrow$ charmonium $+$ charmonium)

Outline

- Soft and Collinear Gluons
- Leading Regions of Feynman Diagrams
- A Previously Unnoticed Complication from Low-Energy Collinear Gluons
- Tools for Proving Factorization
- Factorization of the Soft and Collinear Singularities
- Cancellation of Low-Energy Collinear Singularities
- Cancellation of Soft Singularities
- Redefinition of the Jet and Hard Functions
- NRQCD Factorization

Soft and Collinear Gluons

- For hard-scattering processes in gauge theories, low-virtuality contributions can arise from soft gluons and collinear gluons.

Soft Gluons

- Soft (S) gluon momenta scale as

$$\begin{aligned} k_S &\sim Q\epsilon_S(1, 1, \mathbf{1}_\perp), \\ \epsilon_S &\ll 1. \end{aligned}$$

- There is a soft, logarithmic singularity associated with the limit $\epsilon_S \to 0$.

Collinear Gluons

- Take the momenta of the external mesons to lie approximately along the $+$ and $-$ light-cone directions.
- Collinear ($C^{\pm}$) gluon momenta scale as

$$
\begin{aligned}
k_{C+} &\sim Q\epsilon^{+}[1,(\eta^{+})^{2},\boldsymbol{\eta}_{\perp}^{+}],\\
k_{C-} &\sim Q\epsilon^{-}[(\eta^{-})^{2},1,\boldsymbol{\eta}_{\perp}^{-}],\\
\eta^{\pm} &\ll 1.
\end{aligned}
$$

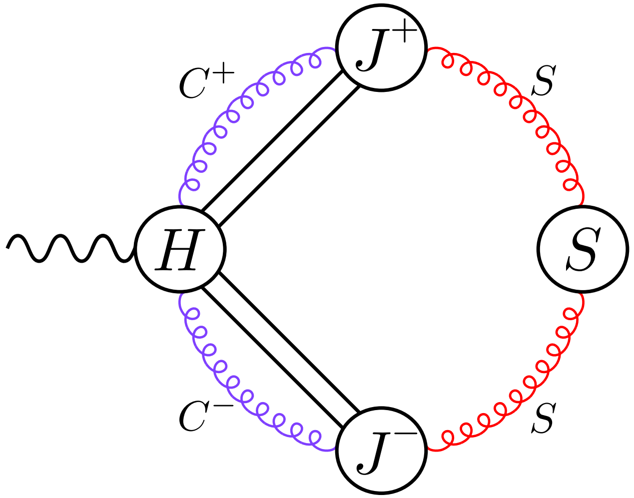
- For massless particles, there is a collinear logarithmic singularity that is associated with the limit $\eta^{\pm} \longrightarrow 0$.
- There is also a soft logarithmic singularity that is associated with the limit $\epsilon^{\pm} \longrightarrow 0$.
- For heavy quarks, the quark mass m protects the amplitude from divergences in the collinear limit.
  - However, we consider would-be collinear divergences that appear in the limit $m \rightarrow 0$.
  - Allows us to organize low-virtuality logarithms associated with the would-be collinear singularities.

Leading Regions of Feynman Diagrams

- Leading regions are the Feynman-diagram topologies that yield singularities that are leading in powers of Q.

- Libby and Sterman (1978) and Collins, Soper, and Sterman (1989):
  One can find the leading regions for gauge theories by analyzing pinch singularities in the momentum contours of integration and by making use of power-counting arguments.

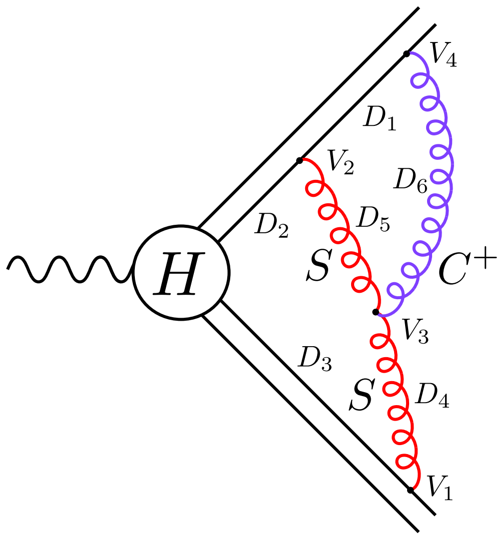
- Work in the Feynman gauge.
- The conventional leading regions have the form



- $J^\pm$ are jet subdiagrams, which contain the external mesons and associated collinear gluons.
- S is a soft subdiagram, which contains soft gluons.
- H is a hard subdiagram, which contains only propagators with virtuality of order Q^2.
- In the conventional picture of the leading regions, soft gluons attach to the collinear subdiagrams and collinear gluons attach only to the hard subdiagram.

A Previously Unnoticed Complication from Low-Energy Collinear Gluons

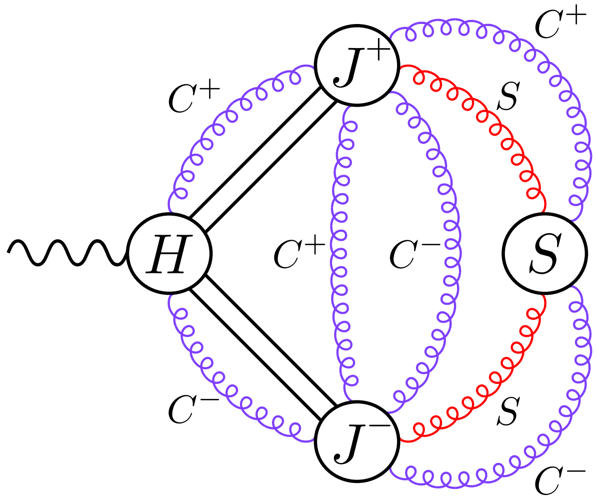
- Low-energy collinear gluons can couple to soft gluons.
- Consider a two-loop example in which a C^+ gluon attaches to a soft gluon:



$$
\begin{array}{rcl}
\text{volume of integration} & \sim & Q^8 \epsilon_S^4 (\epsilon^+)^4 (\eta^+)^4 \\
V_1 \cdot V_2 & \sim & Q^2 \\
V_3 \cdot V_4 & \sim & \epsilon_S Q^2 \\
D_1 & \sim & 1/[Q^2 \epsilon^+ (\eta^+)^2] \\
D_2 & \sim & 1/(Q^2 \epsilon_S) \\
D_3 & \sim & 1/(Q^2 \epsilon_S) \\
D_4 & \sim & 1/(Q^2 \epsilon_S^2) \\
D_5 & \sim & 1/[Q^2 (\epsilon_S^2 + \epsilon_S \epsilon^+)] \\
D_6 & \sim & 1/[Q^2 (\epsilon^+)^2 (\eta^+)^2]
\end{array}
$$

- Gives a contribution that is leading in Q if $\epsilon^+ \sim \epsilon_S$.
- Hence, the leading regions must include couplings of collinear gluons to soft gluons.
- Power-counting arguments also show that low-energy collinear gluons can couple to each other, as well as to the hard subdiagram.

- The neglect of low-energy collinear gluons was a loop-hole in all existing proofs of factorization for inclusive and exclusive processes.
  Discussed for exclusive light-meson production in G. T. Bodwin, X. Garcia i Tormo and J. Lee, Phys. Rev. D **81**, 114005 (2010).
- It follows that the leading regions are more complicated than previously thought:



Tools for Proving Factorization

Collinear Approximations

[GTB (1984); Collins, Sterman, and Soper (1985).]

- In the collinear-gluon propagator numerator, make the replacement

$$g_{\mu\nu} \implies \begin{cases} \dfrac{k_\mu n^-_\nu}{k \cdot n^-} & (C^+), \\ \dfrac{k_\mu n^+_\nu}{k \cdot n^+} & (C^-). \end{cases}$$

  – n^- and n^+ are light-like vectors in the $-$ and $+$ directions, respectively.

- The approximations are exact at the collinear singularities.
- The index μ must attach to a non-$C^\pm$ line.
- The collinear approximations are proportional to k_μ.
  Longitudinal gluon polarization (pure gauge).

Soft Approximation

[Grammer and Yennie (1973); Collins, Sterman, and Soper (1981).]

- In the soft-gluon propagator numerator, make the replacement

$$g_{\mu\nu} \Longrightarrow \frac{k_\mu p_\nu}{k \cdot p}.$$

  - The index μ attaches to a line with momentum p.

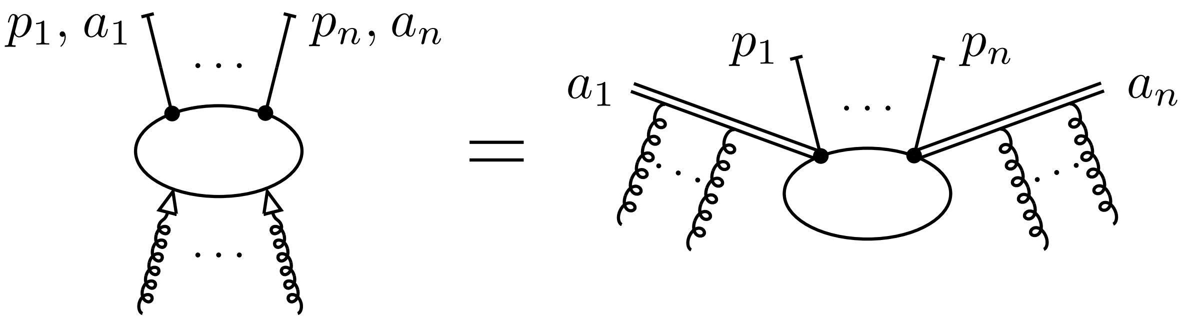
- The soft approximation is exact at the soft singularity.

- The soft approximation is proportional to k_μ.
  Longitudinal gluon polarization (pure gauge).

Decoupling Relations

[Collins and Soper (1981); GTB (1984); Collins, Sterman, and Soper (1985).]

- For longitudinally polarized gluons of the same type (S, C^+, C^-), the graphical Ward-Takahashi lead to a decoupling relation:



- The arrows represents the gluon-propagator numerator factors from the soft, C^+, or C^- approximation.

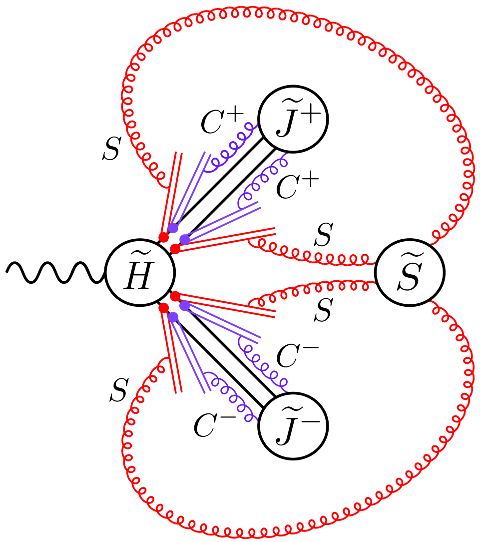
- The “‘eikonal” (double) lines have
  - vertices of the form n^-_μ,n^+_μ, or p_μ,
  - propagators of the form $1/(k \cdot n^-)$, $1/(k \cdot n^+)$, or $1/(k \cdot p)$.

  - The eikonal lines are path-ordered exponentials of path integrals of gauge fields.

Factorization of the Soft and Collinear Singularities

- In analyzing the soft and collinear singularities, we need to consider the possibility that different gluons can approach the soft and collinear limits at different rates.
  - Power-counting arguments show that the exterior divergences “control” the interior divergences.
- Follow an iterative procedure, starting with the singularities that are innermost in the Feynman diagrams and working to the outside.
- Apply the soft and collinear approximations and the decoupling relations at each stage.
- New eikonal-line identities can be used to combine contributions from successive stages.

- The result is that the soft and collinear singular contributions decouple:



$\tilde{S}$, $\tilde{J}^{\pm}$ denote the singular parts of S and $J^{\pm}$.

Cancellation of Soft Singularities

- The soft eikonal lines that attach to a quark and an antiquark in a given meson cancel.
- They run in opposite directions (up to corrections of order mv/Q).
- They end on space-time points that are separated by $k_\mu/Q \to 0$.
- The cancellation relies on the color-singlet nature of the quarkonium.

Cancellation of Low-Energy Collinear Singularities

- There is also a cancellation of the parts of the quark and antiquark collinear eikonal lines for which the energies of the collinear gluons are much less than Q.
- Implies, that the couplings of the low-energy $C^{\pm}$ gluons to subdiagrams outside $J^{\pm}$ cancel, but only after considerable re-organization.

Redefinition of the Jet and Hard Functions

- Now we can extend the ranges of integration in $\tilde{J}^{\pm}$ up to an ultraviolet cutoff $\mu_F \sim Q$, which acts as a factorization scale.
  Incorporates collinear logarithms into $\tilde{J}^{\pm}$.
- Re-define $\tilde{H}$ to be the complete amplitude divided by $\tilde{J}^{+}$ and $\tilde{J}^{-}$.

- $\tilde{H}$ is free of soft and would-be collinear singularities and their associated logarithms.
- The amplitude for $e^+e^- \to$ charmonium $+$ charmonium now has the form

$$A = \tilde{J}^{-} \otimes \tilde{H} \otimes \tilde{J}^{+},$$

- $\tilde{H}$ contains only virtualities of order Q^2 and $\tilde{J}^{+}$ and $\tilde{J}^{-}$ contain all of the collinear contributions with virtualities $\ll Q$.

NRQCD Factorization

- A further factorization of $\tilde{J}^{+}$ and $\tilde{J}^{-}$ into products of NRQCD matrix elements and short-distance coefficients leads to the stated factorized form.

Other Activities

- Convener, Production Section of the Quarkonium Working Group, 2002–present
- Convener, Quarkonium Working Group, 2005–present
- Proposer and organizer, Kavli Institute on Effective Field Theories, Beijing, August 3–September 11, 2009
- Co-chair of the local organizing committee, 7th International Workshop on Heavy Quarkonium, FNAL, May 18–21, 2010
- Convener, Heavy Quarks Section, International Conference on Quark Confinement and the Hadron Spectrum, Madrid, August 30–September 3, 2010

Backup Slides

Questions

Why Do We Need to Consider Gluons with Energy Less Than $\Lambda_{\rm QCD}$?

- Because of color confinement, gluons with momentum components less than of order $\Lambda_{\rm QCD}$ are unphysical.
- However, as we have seen, low-energy gluons can appear in perturbation theory in leading power in Q, for example, in calculations of short-distance coefficients.
- In order to establish the consistency of perturbative calculations of short-distance coefficients, it is necessary to prove that the contributions from low-energy gluons can be re-organized into the standard factorized form.

Why can't one treat the low-energy collinear gluons in the soft approximation?

- Then they would be accounted for in the soft (usoft) part of the SCET action.
- The problem is that the soft approximation becomes singular for collinear gluons.
- Let the momenta of a quark and antiquark in a meson be p_q and $p_{\bar{q}}$, with

$$p_q = \left[zQ, \frac{\boldsymbol{p}_\perp^2}{2zQ}, \boldsymbol{p}_\perp \right]; \quad p_{\bar{q}} = \left[(1-z)Q, \frac{\boldsymbol{p}_\perp^2}{2(1-z)Q}, -\boldsymbol{p}_\perp \right]; \quad \boldsymbol{p}_\perp \sim \Lambda_{\text{QCD}}.$$

- The soft approximations for the quark and antiquark lines are

$$g^{\mu\nu} \Longrightarrow \begin{cases} \dfrac{k^\mu p_q^\nu}{k \cdot p_q} & \text{for the quark;} \\ \dfrac{k^\mu p_{\bar{q}}^\nu}{k \cdot p_{\bar{q}}} & \text{for the antiquark.} \end{cases}$$

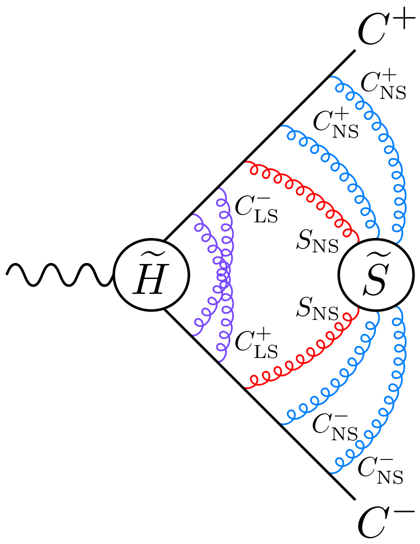
- When k is collinear to p_q ($p_{\bar{q}}$), the quark (antiquark) soft approximation becomes infinite and the cancellation between the quark and antiquark contributions fails.
  The soft subdiagram fails to decouple.
- For soft gluons, the cancellation holds up to corrections of order Λ_{QCD}/Q, but for collinear gluons, the cancellation fails whenever $\boldsymbol{p}_\perp$ is nonvanishing.

The contributions from low-energy gluons have scaleless integrands. Why don't they simply vanish in dimens

- Integrals with scaleless integrands vanish when the limits of integration are 0 or $\pm\infty$.
  - Extending the range of integration to infinity, as in the method of regions, introduces the possibility of double counting.
  - There is no proof that such a procedure is correct.
  - In the end, the contributions from the low-energy gluons cancel, but only after considerable re-arrangement.
  - The cancellation does not rely the range of integration being infinite.
  - In our proof, there is no double counting issue because we only need to show that the soft and collinear singularities decouple.

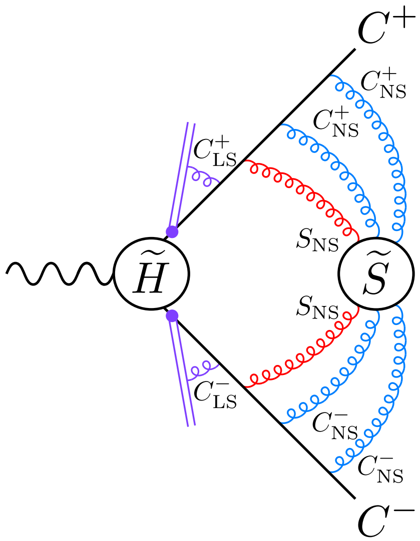
Factorization at Each Energy Level

- At each energy level, we need to consider
  - soft gluons with energy of a nominal scale (NS),
  - collinear gluons with energy of the nominal scale (NS),
  - collinear gluons with energy of the large scale (LS).
  - The LS is much larger than the NS, but much smaller than the NS of the next larger (inner) level.
- At the highest energy level, we have a configuration of the form

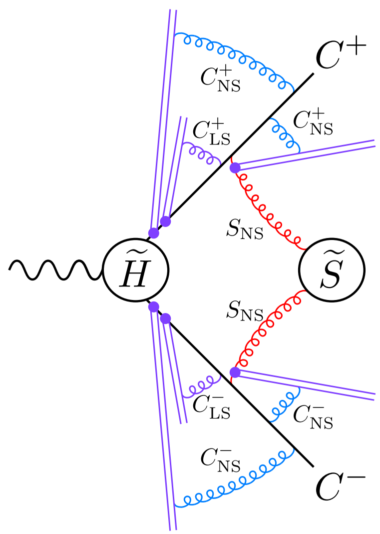


  - We have suppressed
    - the antiquark lines in each meson,
    - gluons at lower levels, which lie to the outside of the gluons that are shown.

- Use the collinear approximations and decoupling relations to decouple the LS collinear gluons:

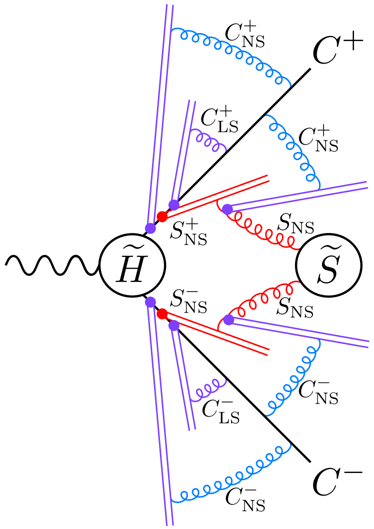


- Use the collinear approximations and decoupling relations to decouple partially the NS collinear gluons:

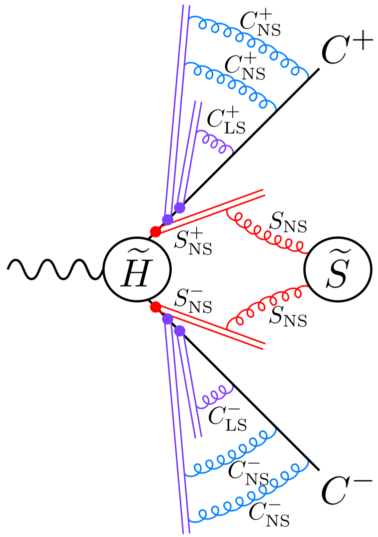


  – The NS collinear gluons have eikonal lines that attach to the soft NS gluons.

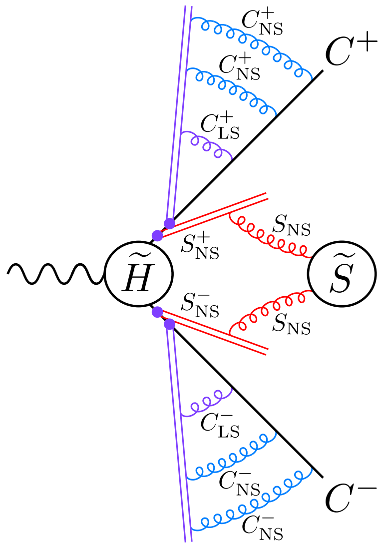
- Use the soft approximation and decoupling relation to decouple partially the soft gluons, along with the attached collinear eikonal lines:



- Use relationships between the NS collinear eikonal lines to recombine them:



- Combine the NS and LS collinear eikonal lines:



- This same procedure can be used at the next lower (outer) level.
  - The new soft and collinear eikonal lines that appear at each level can be combined with the soft and collinear eikonal lines that appeared at the previous level.

- Proceeding iteratively, we can achieve the factorized for all levels: